ECE 312
Electronic Circuits (A)

Lec. 6: BJT Modeling and re Transistor Model (small signal analysis) (1)

Instructor

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Agenda

- Amplification in the AC Domain
- BJT transistor Modeling
- The r_e Transistor Model (small signal analysis) (1)

Amplification in the AC Domain

Amplification in the AC Domain

$$\eta = P_o/P_i \text{ cannot be greater than 1.}$$

In fact, a *conversion efficiency* is defined by $\eta = P_{o(\mathrm{ac})}/P_{i(\mathrm{dc})}$, where $P_{o(\mathrm{ac})}$ is the ac power to the load and $P_{i(\mathrm{dc})}$ is the dc power supplied.

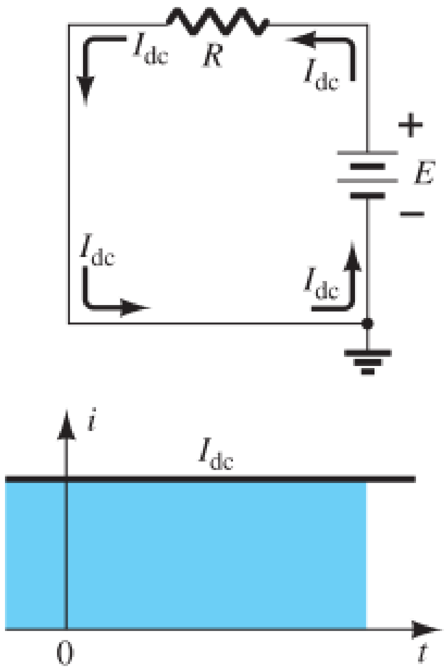


FIG. 5.1
Steady current established by a dc supply.

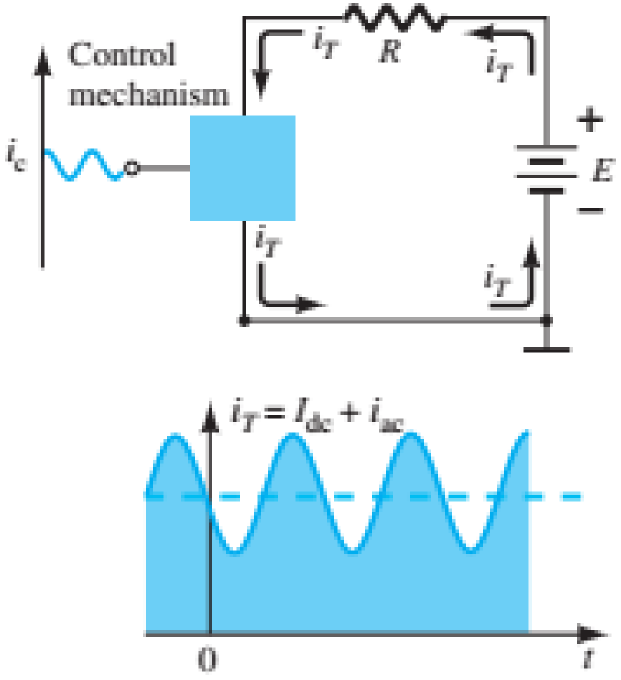
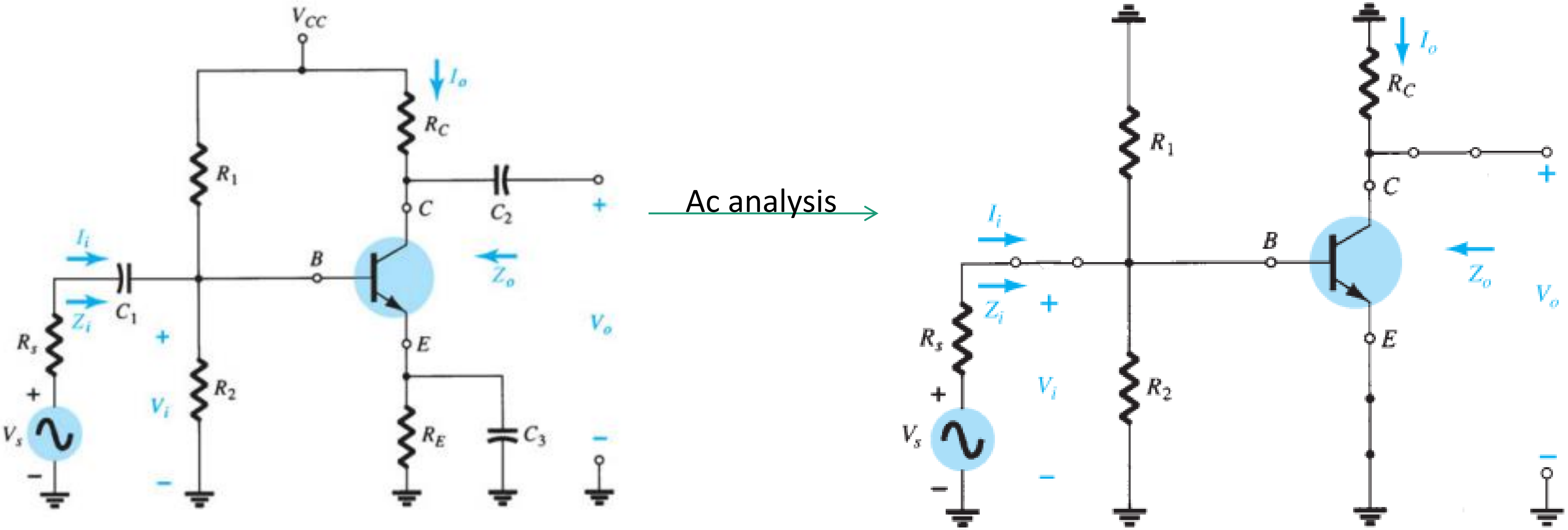


FIG. 5.2
Effect of a control element on the steady-state flow of the electrical system of Fig. 5.1.

The superposition theorem is applicable for the analysis and design of the DC and AC components of a BJT network, permitting the separation of the analysis of the dc and ac responses of the system.

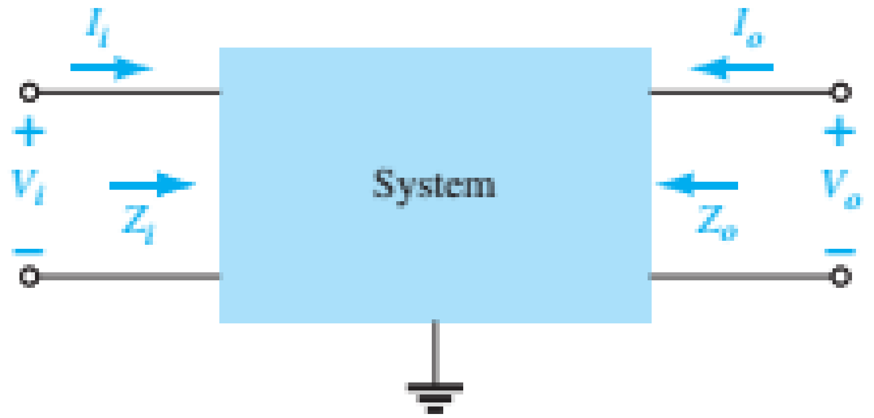
BJT Transistor Modeling

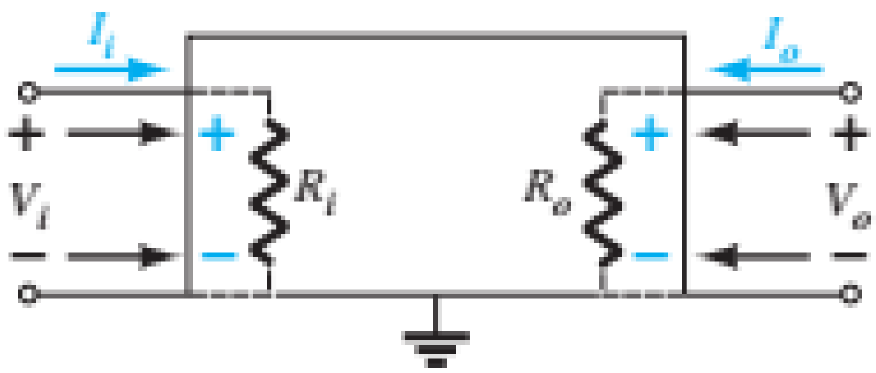
BJT Transistor Modeling



BJT Transistor Modeling (1 of 2)

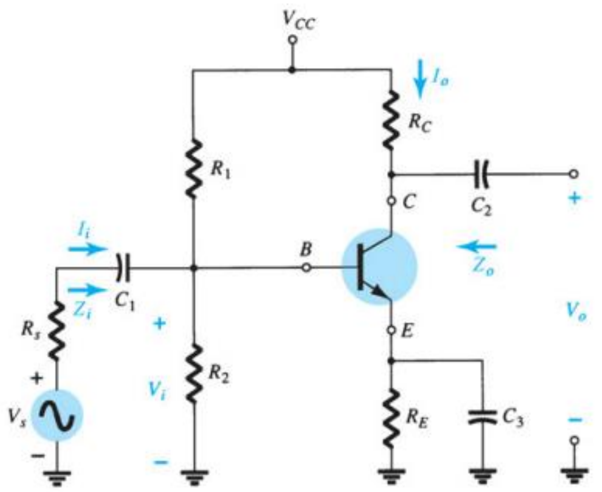
- A model is a combination of circuit elements, properly chosen, that best approximates the actual behavior of a semiconductor device under specific operating conditions.
- Any electronic system has some important parameters have to be determined
  - Input and Output Voltage
  - Input and Output Impedance
  - Input and Output Current

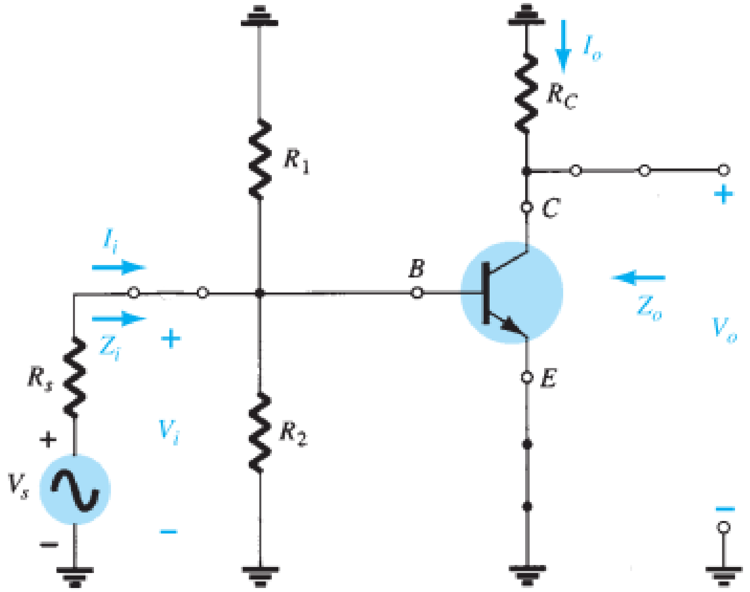


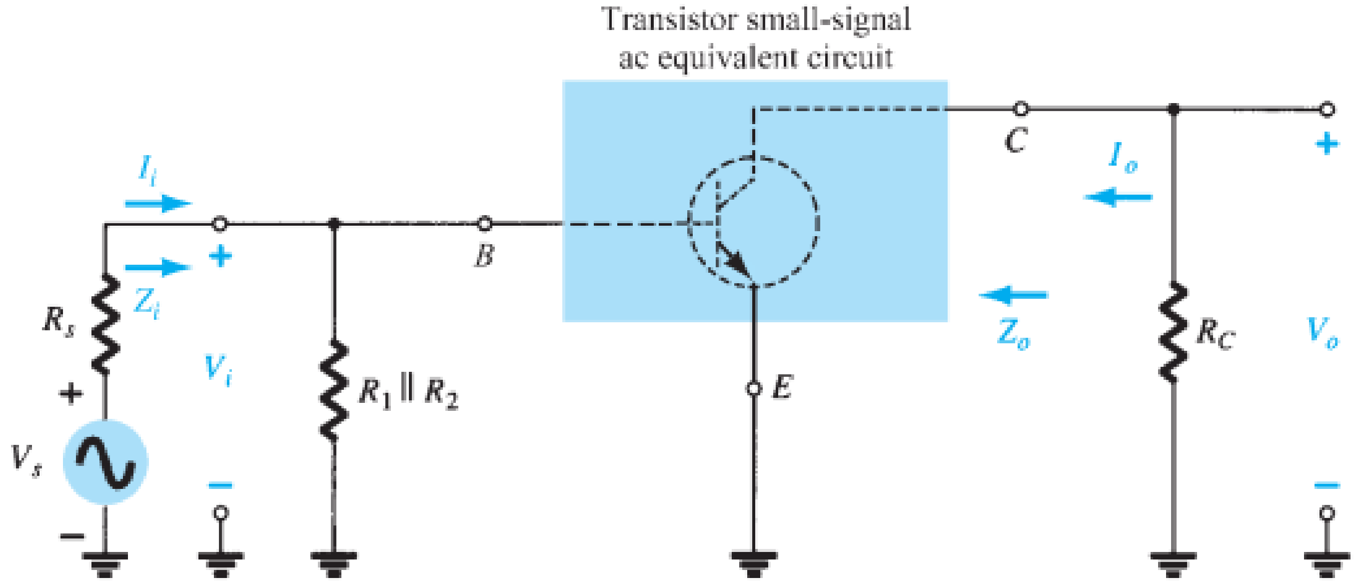


BJT Transistor Modeling (2 of 2)

- The ac equivalent of a transistor network is obtained by:
    1. Setting all dc sources to zero and replacing them by a short-circuit equivalent
    2. Replacing all capacitors by a short-circuit equivalent
    3. Removing all elements bypassed by the short-circuit equivalents introduced by steps 1 and 2
    4. Redrawing the network in a more convenient and logical form







The r_e Transistor Model

- Common Emitter Configuration
- Common Base Configuration
- Common Collector Configuration
- r_e Model in Different Bias Circuits

The r_e Transistor Model (CE)

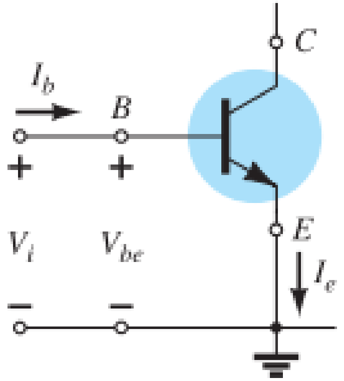


FIG. 5.8
Finding the input equivalent circuit for a BJT transistor.

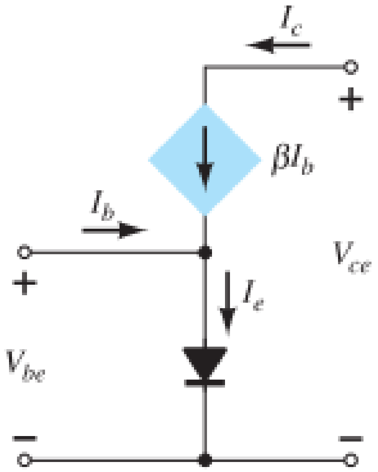


FIG. 5.12
BJT equivalent circuit.

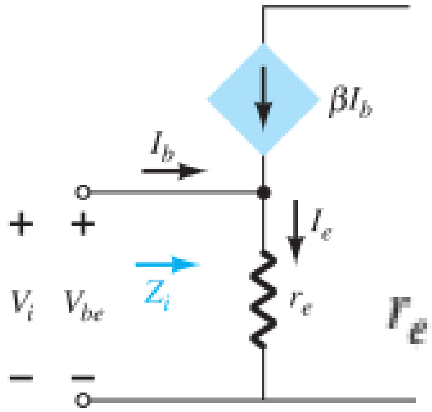


FIG. 5.13
Defining the level of Z_i.

$$r_e = 26\,\text{mV}/I_E$$

$$Z_i = \frac{V_i}{I_b} = \frac{V_{be}}{I_b}$$

$$V_{be} = I_e r_e = (I_c + I_b)r_e = (\beta I_b + I_b)r_e$$
$$= (\beta + 1)I_b r_e$$

$$Z_i = \frac{V_{be}}{I_b} = \frac{(\beta + 1)I_b r_e}{I_b}$$

$$\boxed{Z_i = (\beta + 1)r_e \cong \beta r_e}$$

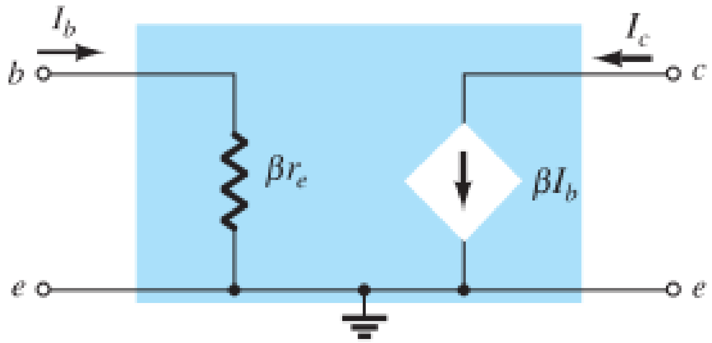


FIG. 5.14
Improved BJT equivalent circuit.

The r_e Transistor Model (CE)

Early Voltage

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta I_C}{\Delta V_{CE}} = \frac{1}{r_o}$$

$$\boxed{r_o = \frac{\Delta V_{CE}}{\Delta I_C}}$$

$$\boxed{r_o = \frac{\Delta V}{\Delta I} = \frac{V_A + V_{CE_Q}}{I_{C_Q}}}$$

$$\boxed{r_o \cong \frac{V_A}{I_{C_Q}}}$$

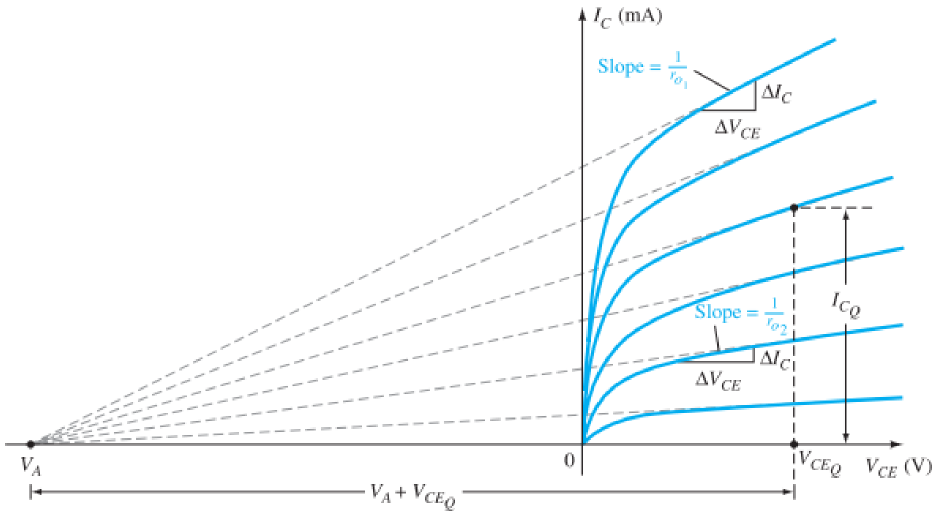


FIG. 5.15
Defining the Early voltage and the output impedance of a transistor.

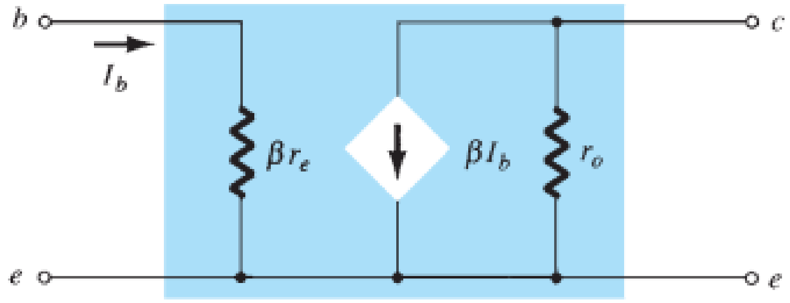


FIG. 5.16
r_e model for the common-emitter transistor configuration including effects of r_o.

The r_e Transistor Model (CB)

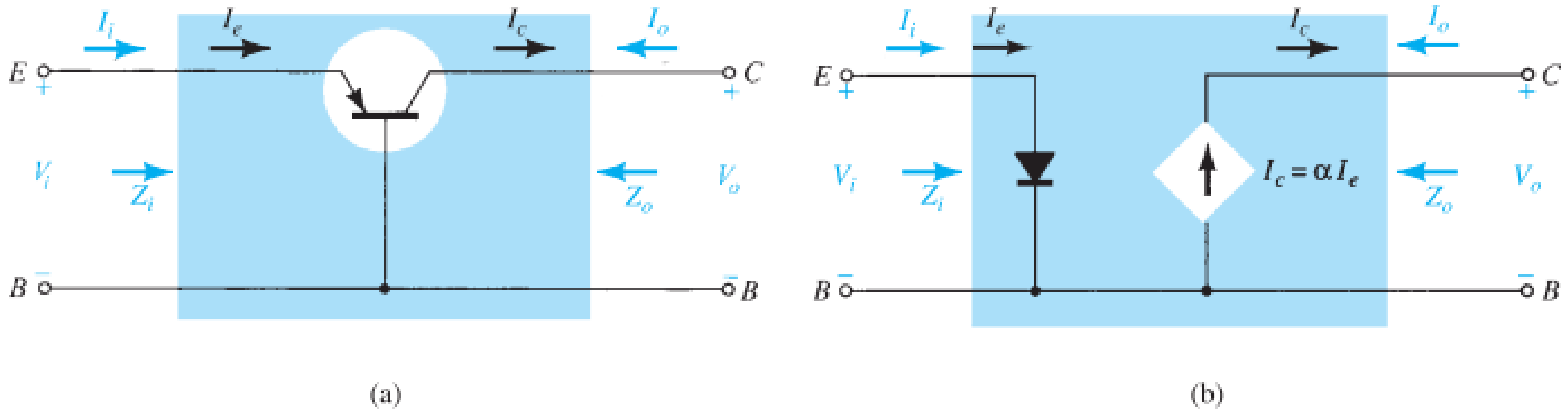


FIG. 5.17
(a) Common-base BJT transistor; (b) equivalent circuit for configuration of (a).

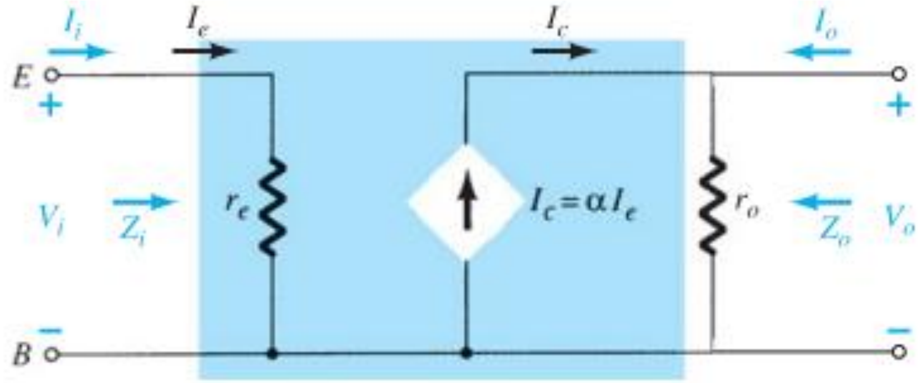


FIG. 5.18
Common base r_e equivalent circuit.

The r_e Transistor Model (CC)

- For the common-collector configuration, the model defined for the common-emitter configuration of is normally applied rather than defining a model for the common-collector configuration.

npn versus pnp

- The dc analysis of *npn* and *pnp* configurations is quite different in the sense that the currents will have opposite directions and the voltages opposite polarities.
- However, for an ac analysis where the signal will progress between positive and negative values, the ac equivalent circuit will be the same.

C.E. Fixed Bias Configuration

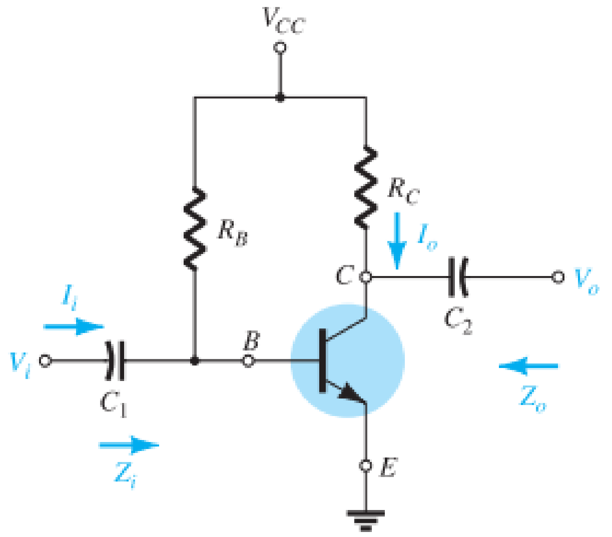


FIG. 5.20
Common-emitter fixed-bias configuration.

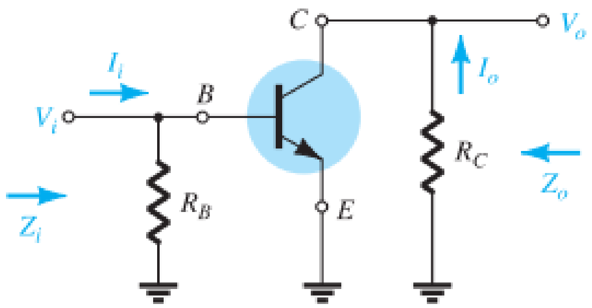


FIG. 5.21
Network of Fig. 5.20 following the removal of the effects of V_{CC}, C_1, and C_2.

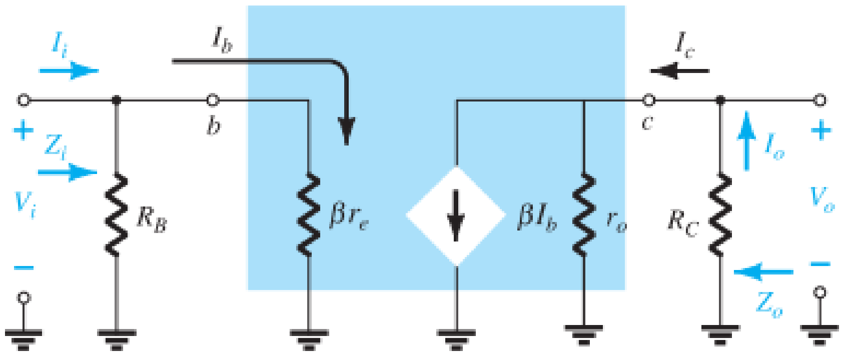


FIG. 5.22
Substituting the r_e model into the network of Fig. 5.21.

$\boxed{Z_i = R_B \| \beta r_e}$ ohms

$\boxed{Z_i \cong \beta r_e}$ $_{R_B \geq 10\beta r_e}$ ohms

$\boxed{Z_o = R_C \| r_o}$ ohms

$\boxed{Z_o \cong R_C}$ $_{r_o \geq 10 R_C}$

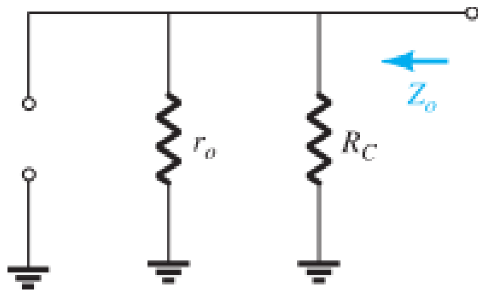


FIG. 5.23
Determining Z_o for the network of Fig. 5.22.

$$V_o = -\beta I_b (R_C \| r_o)$$

$$I_b = \frac{V_i}{\beta r_e}$$

$$V_o = -\beta \left(\frac{V_i}{\beta r_e}\right)(R_C \| r_o)$$

$$\boxed{A_v = \frac{V_o}{V_i} = -\frac{(R_C \| r_o)}{r_e}}$$

$$\boxed{A_v = -\frac{R_C}{r_e}} \quad {}_{r_o \geq 10 R_C}$$

C.E. Fixed Bias Configuration (Phase relationship)

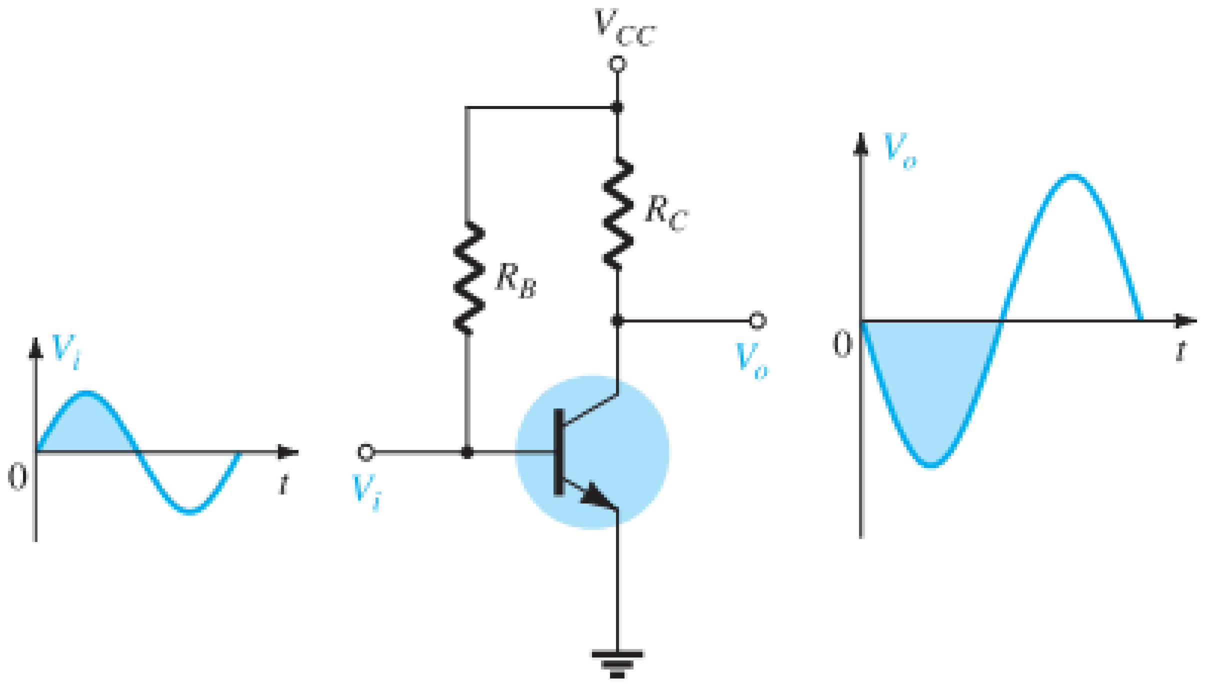


FIG. 5.24
Demonstrating the 180° phase shift between input and output waveforms.

C.E. Fixed Bias Configuration (Example)

EXAMPLE 5.1 For the network of Fig. 5.25:

a. Determine r_e.
b. Find Z_i (with $r_o = \infty\,\Omega$).
c. Calculate Z_o (with $r_o = \infty\,\Omega$).
d. Determine A_v (with $r_o = \infty\,\Omega$).
e. Repeat parts (c) and (d) including $r_o = 50\,\text{k}\Omega$ in all calculations and compare results.

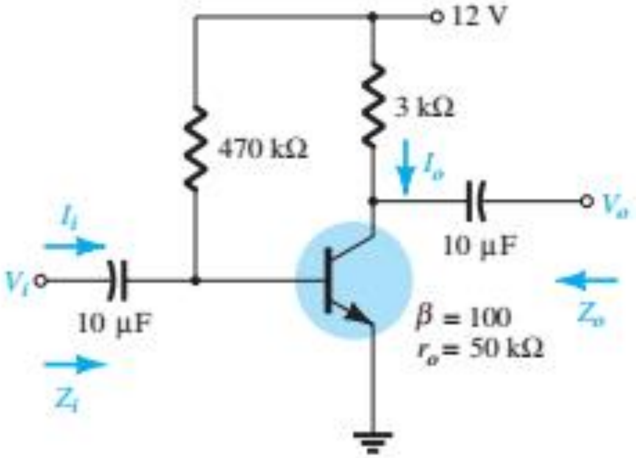


FIG. 5.25
Example 5.1.

Solution:

a. DC analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega} = 24.04\,\mu\text{A}$$

$$I_E = (\beta + 1)I_B = (101)(24.04\,\mu\text{A}) = 2.428\text{ mA}$$

$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{2.428\text{ mA}} = \mathbf{10.71\,\Omega}$$

b. $\beta r_e = (100)(10.71\,\Omega) = 1.071\text{ k}\Omega$

$$Z_i = R_B \| \beta r_e = 470\text{ k}\Omega \| 1.071\text{ k}\Omega = \mathbf{1.07\text{ k}\Omega}$$

c. $Z_o = R_C = \mathbf{3\text{ k}\Omega}$

d. $A_v = -\dfrac{R_C}{r_e} = -\dfrac{3\text{ k}\Omega}{10.71\,\Omega} = \mathbf{-280.11}$

e. $Z_o = r_o \| R_C = 50\text{ k}\Omega \| 3\text{ k}\Omega = \mathbf{2.83\text{ k}\Omega}$ vs. $3\text{ k}\Omega$

$$A_v = -\frac{r_o \| R_C}{r_e} = \frac{2.83\text{ k}\Omega}{10.71\,\Omega} = \mathbf{-264.24}\text{ vs. } -280.11$$

C.E. Voltage-Divider Bias

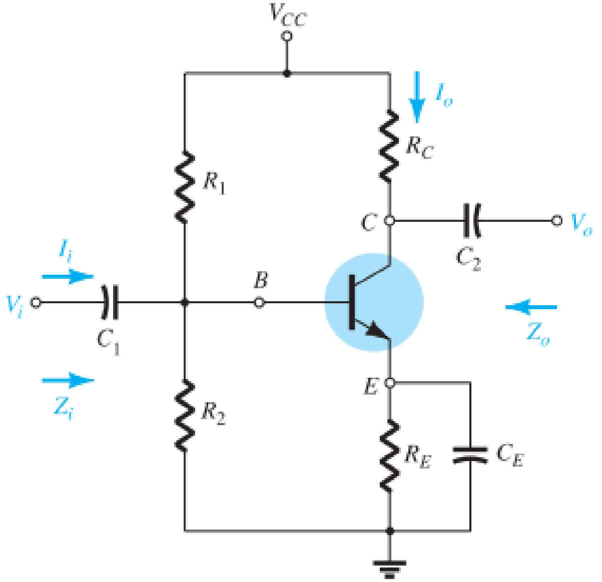


FIG. 5.26
Voltage-divider bias configuration.

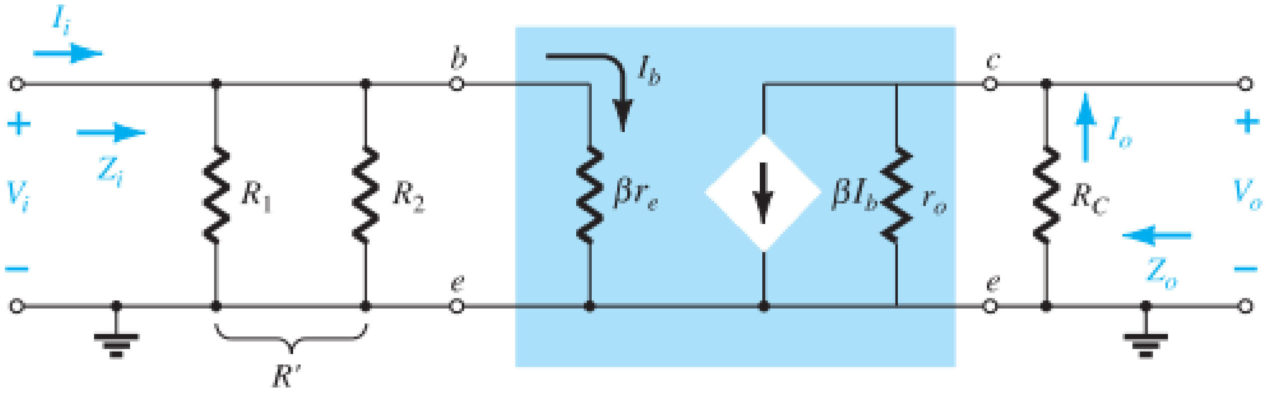


FIG. 5.27
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.26.

$$\boxed{R' = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}}$$

$$\boxed{Z_i = R' \| \beta r_e}$$

$$\boxed{Z_o = R_C \| r_o}$$

$$\boxed{Z_o \cong R_C} \quad {}_{r_o \geq 10R_C}$$

$$V_o = -(\beta I_b)(R_C \| r_o)$$

$$I_b = \frac{V_i}{\beta r_e}$$

$$V_o = -\beta\left(\frac{V_i}{\beta r_e}\right)(R_C \| r_o)$$

$$\boxed{A_v = \frac{V_o}{V_i} = \frac{-R_C \| r_o}{r_e}}$$

$$\boxed{A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{r_e}} \quad {}_{r_o \geq 10R_C}$$

180° phase shift

C.E. Emitter Bias Configuration (Un-bypassed without ro)

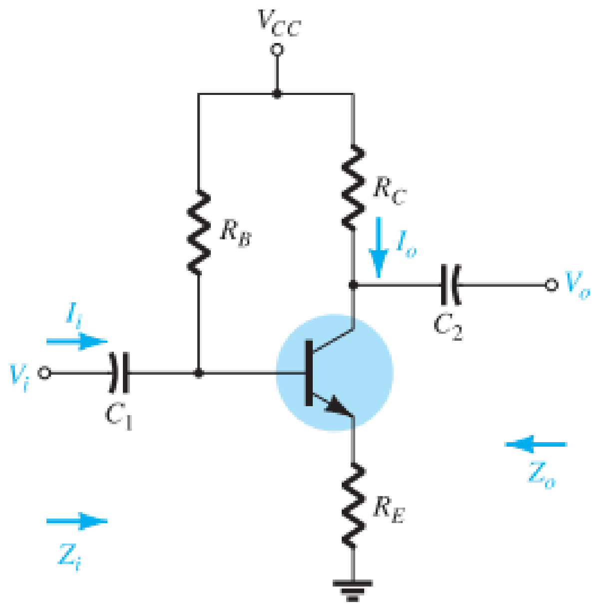


FIG. 5.29
CE emitter-bias configuration.

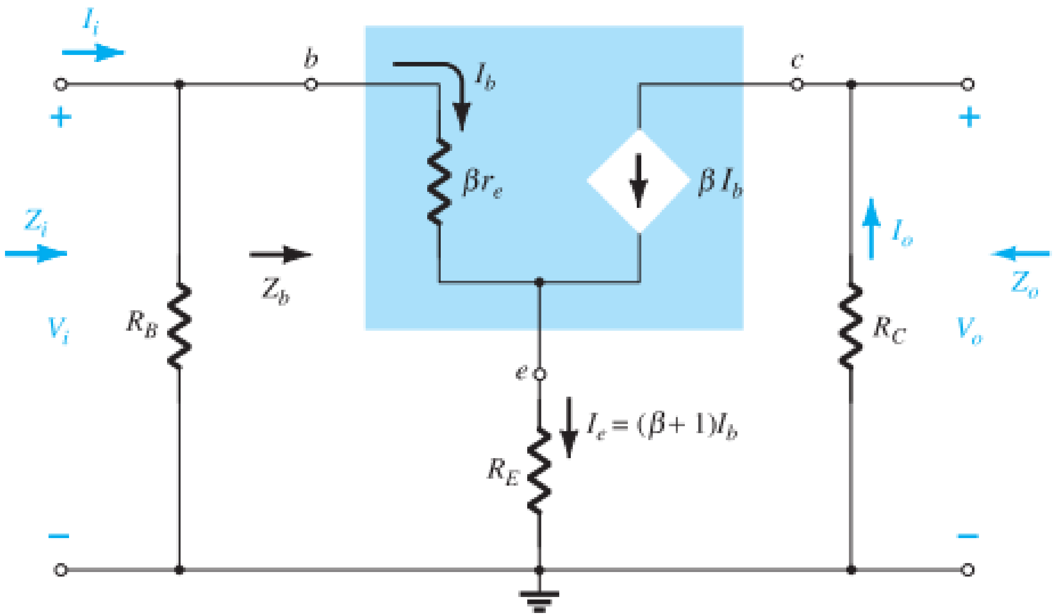


FIG. 5.30
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.29.

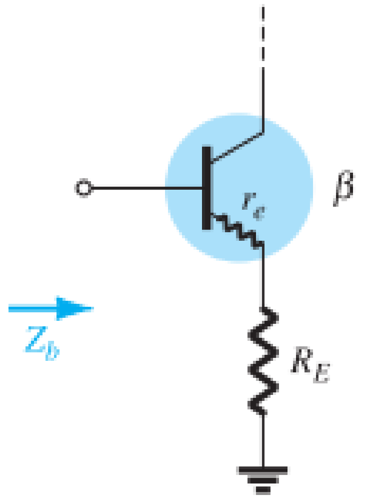


FIG. 5.31
Defining the input impedance of a transistor with an unbypassed emitter resistor.

$$V_i = I_b\beta r_e + I_e R_E$$

$$V_i = I_b\beta r_e + (\beta + 1)I_b R_E$$

$$Z_b = \frac{V_i}{I_b} = \beta r_e + (\beta + 1)R_E$$

$$\boxed{Z_b = \beta r_e + (\beta + 1)R_E}$$

$$Z_b \cong \beta r_e + \beta R_E$$

$$\boxed{Z_b \cong \beta(r_e + R_E)}$$

$$\boxed{Z_b \cong \beta R_E}$$

$$\boxed{Z_i = R_B \| Z_b}$$

$$\boxed{Z_o = R_C}$$

C.E. Emitter Bias Configuration (Un-bypassed without ro)

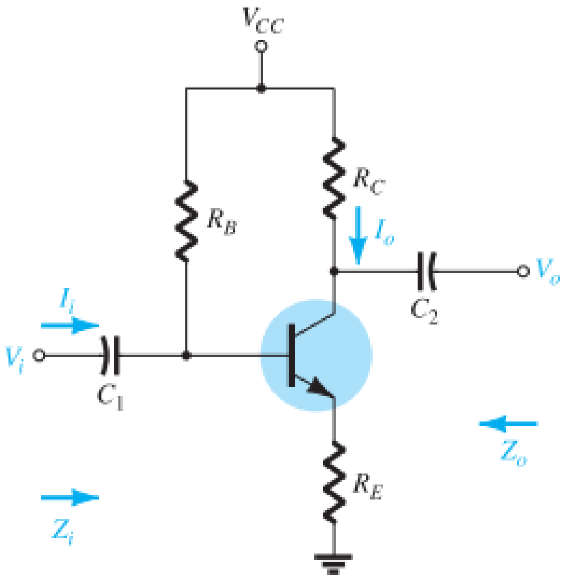


FIG. 5.29
CE emitter-bias configuration.

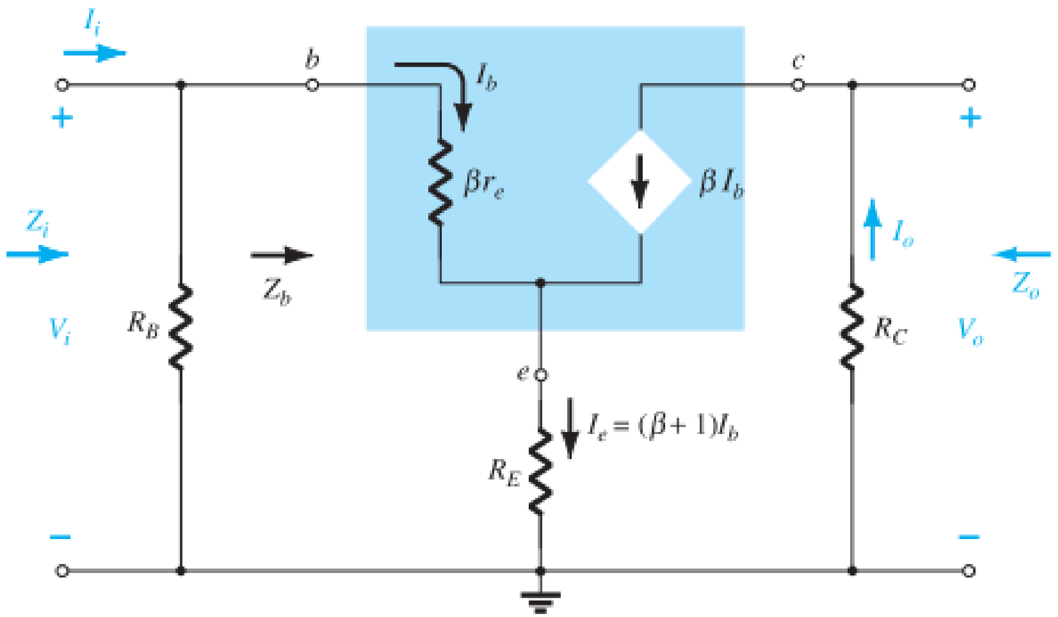


FIG. 5.30
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.29.

$$I_b = \frac{V_i}{Z_b}$$

$$V_o = -I_o R_C = -\beta I_b R_C$$

$$= -\beta\left(\frac{V_i}{Z_b}\right)R_C$$

$$\boxed{A_v = \frac{V_o}{V_i} = -\frac{\beta R_C}{Z_b}}$$

$$Z_b \cong \beta(r_e + R_E)$$

$$\boxed{A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{r_e + R_E}}$$

$$Z_b \cong \beta R_E$$

$$\boxed{A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{R_E}}$$

180° phase shift

C.E. Emitter Bias Configuration (Un-bypassed with ro)

$$\boxed{Z_i = R_B \| Z_b}$$

$$\boxed{Z_b = \beta r_e + \left[\frac{(\beta + 1) + R_C/r_o}{1 + (R_C + R_E)/r_o}\right] R_E}$$

R_C/r_o is always much less than $(\beta + 1)$,

$$Z_b \cong \beta r_e + \frac{(\beta + 1)R_E}{1 + (R_C + R_E)/r_o}$$

For $r_o \geq 10(R_C + R_E)$,

$$Z_b \cong \beta r_e + (\beta + 1)R_E$$

$$\boxed{Z_b \cong \beta(r_e + R_E)}$$

$r_o \geq 10(R_C + R_E)$

$$\boxed{Z_o = R_C \left\| \left[r_o + \frac{\beta(r_o + r_e)}{1 + \dfrac{\beta r_e}{R_E}} \right] \right.}$$

$r_o \gg r_e$,

$$Z_o \cong R_C \left\| r_o \left[1 + \frac{\beta}{1 + \dfrac{\beta r_e}{R_E}} \right] \right.$$

$$Z_o \cong R_C \left\| r_o \left[1 + \frac{1}{\dfrac{1}{\beta} + \dfrac{r_e}{R_E}} \right] \right.$$

Typically $1/\beta$ and r_e/R_E are less than one with a sum usually less than one.

$$\boxed{Z_o \cong R_C}$$

Any level of r_o

$$\boxed{A_v = \frac{V_o}{V_i} = \frac{-\dfrac{\beta R_C}{Z_b}\left[1 + \dfrac{r_e}{r_o}\right] + \dfrac{R_C}{r_o}}{1 + \dfrac{R_C}{r_o}}}$$

$$\frac{r_e}{r_o} \ll 1,$$

$$A_v = \frac{V_o}{V_i} \cong \frac{-\dfrac{\beta R_C}{Z_b} + \dfrac{R_C}{r_o}}{1 + \dfrac{R_C}{r_o}}$$

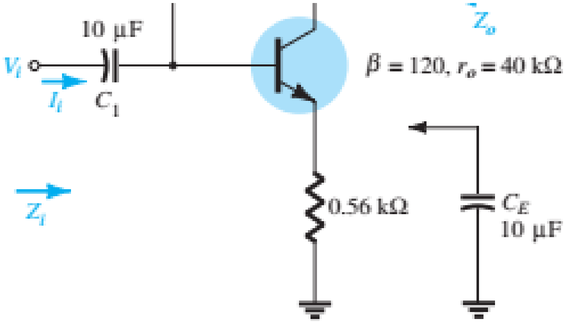
$r_o \geq 10R_C$,

$$\boxed{A_v = \frac{V_o}{V_i} \cong -\frac{\beta R_C}{Z_b}}$$

$r_o \geq 10R_C$

C.E. Emitter Bias Configuration (bypassed)

Same as CE fixed bias config.



Portion bypassed

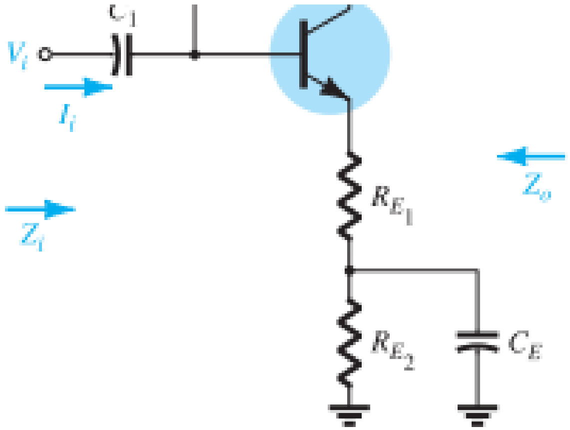


FIG. 5.35

An emitter-bias configuration with a portion of the emitter-bias resistance bypassed in the ac domain.

C.E. Emitter Bias Configuration (Example)

EXAMPLE 5.3 For the network of Fig. 5.32, without C_E (unbypassed), determine:

a. r_e.
b. Z_i.
c. Z_o.
d. A_v.

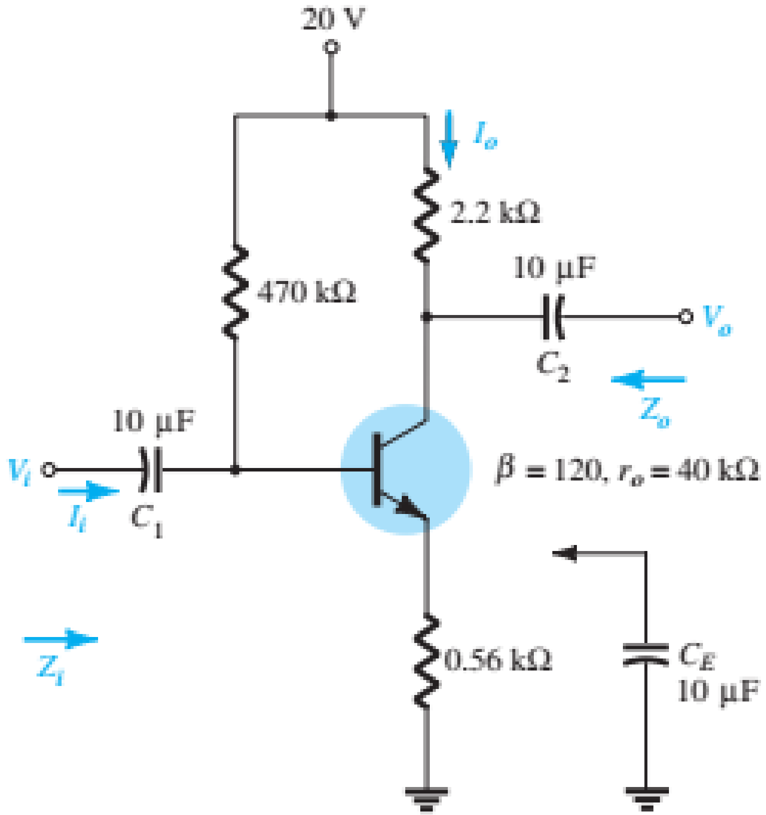


FIG. 5.32
Example 5.3.

Solution:

a. DC:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} = \frac{20\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega + (121)0.56\text{ k}\Omega} = 35.89\ \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (121)(35.89\ \mu\text{A}) = 4.34\text{ mA}$$

and $$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{4.34\text{ mA}} = \mathbf{5.99\ \Omega}$$

b. Testing the condition $r_o \geq 10(R_C + R_E)$, we obtain

$$40\text{ k}\Omega \geq 10(2.2\text{ k}\Omega + 0.56\text{ k}\Omega)$$
$$40\text{ k}\Omega \geq 10(2.76\text{ k}\Omega) = 27.6\text{ k}\Omega\ (\textit{satisfied})$$

Therefore,

$$Z_b \cong \beta(r_e + R_E) = 120(5.99\ \Omega + 560\ \Omega)$$
$$= 67.92\text{ k}\Omega$$

and $$Z_i = R_B \| Z_b = 470\text{ k}\Omega \| 67.92\text{ k}\Omega$$
$$= \mathbf{59.34\text{ k}\Omega}$$

c. $Z_o = R_C = \mathbf{2.2\text{ k}\Omega}$

d. $r_o \geq 10R_C$ is satisfied. Therefore,

$$A_v = \frac{V_o}{V_i} \cong -\frac{\beta R_C}{Z_b} = -\frac{(120)(2.2\text{ k}\Omega)}{67.92\text{ k}\Omega}$$
$$= \mathbf{-3.89}$$

compared to -3.93 using Eq. (5.20): $A_v \cong -R_C/R_E$.

C.E. Emitter Bias Configuration (Example)

EXAMPLE 5.3 For the network of Fig. 5.32, without C_E (unbypassed), determine:

a. r_e.
b. Z_i.
c. Z_o.
d. A_v.

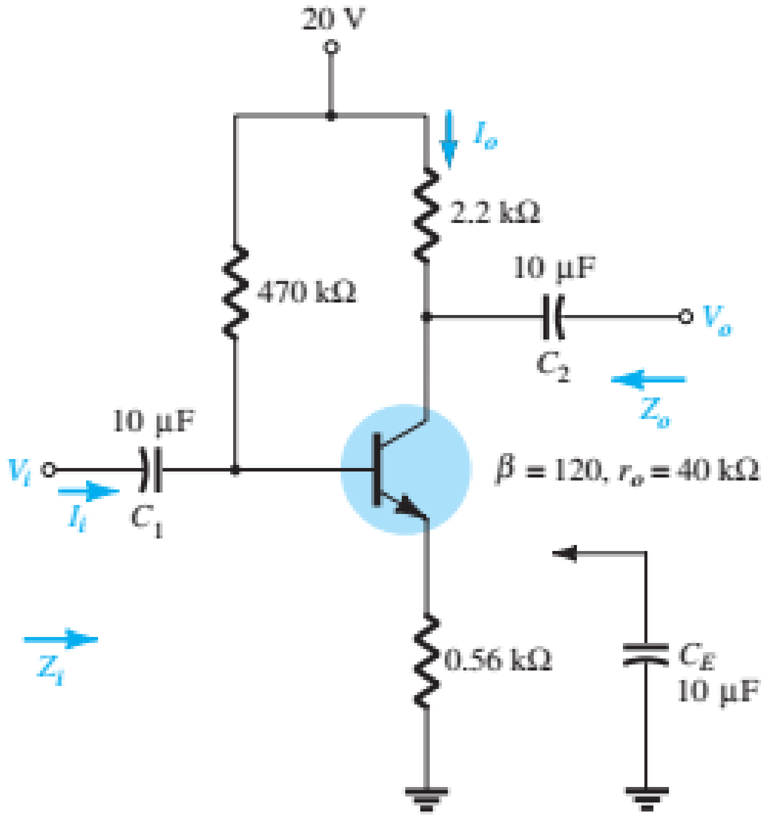


FIG. 5.32
Example 5.3.

EXAMPLE 5.4 Repeat the analysis of Example 5.3 with C_E in place.

Solution:

a. The dc analysis is the same, and $r_e = 5.99\ \Omega$.
b. R_E is "shorted out" by C_E for the ac analysis. Therefore,

$$Z_i = R_B \| Z_b = R_B \| \beta r_e = 470\ \text{k}\Omega \| (120)(5.99\ \Omega)$$
$$= 470\ \text{k}\Omega \| 718.8\ \Omega \cong \mathbf{717.70\ \Omega}$$

c. $Z_o = R_C = \mathbf{2.2\ k\Omega}$

d. $$A_v = -\frac{R_C}{r_e}$$
$$= -\frac{2.2\ \text{k}\Omega}{5.99\ \Omega} = \mathbf{-367.28}\ \text{(a significant increase)}$$


Thank You!